

Article presentation (64)

Srinivasan Ruina Nature 2006 walking - running

Authors :

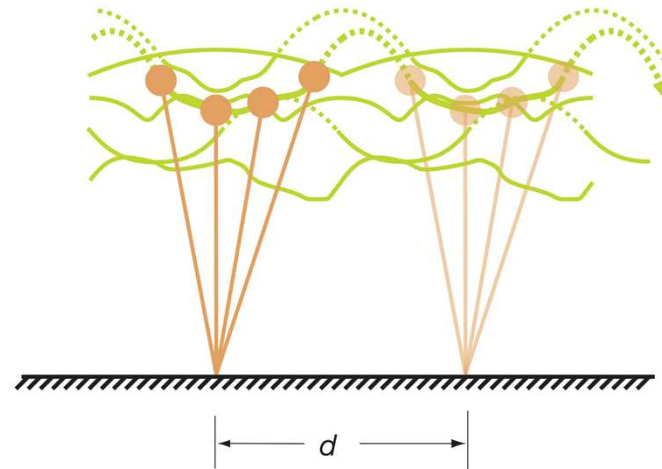
- Ashvin Kugathanan
- Arthur Bauer
- Maxime Luyet

Computer optimization of a minimal biped model discovers walking and running

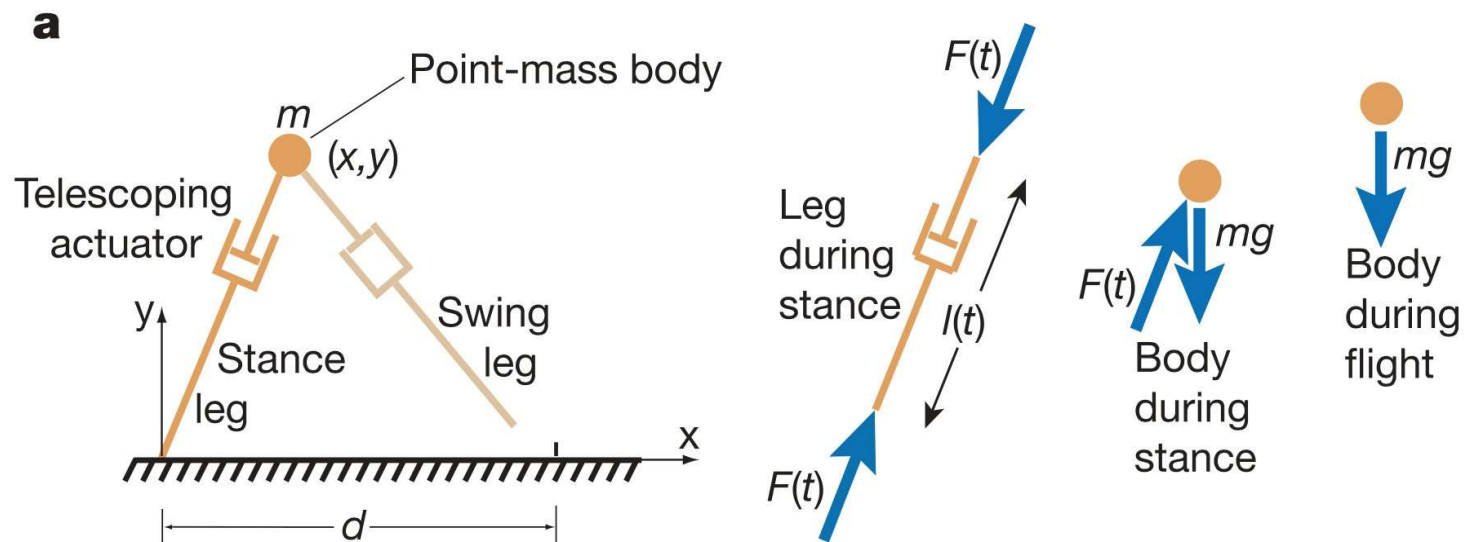
Manoj Srinivasan & Andy Ruina

Why do humans mainly use **two gaits**—walking at low speeds and running at high speeds

- > most energy-efficient gait



Main idea



- **Body = point mass**, no rotation.
- **Massless legs**, no inertia, no swing cost.
- Legs can **change length/orientation with zero energy** when off the ground.
- **Telescopic actuator** applies **only axial compressive force** $F(t)$.
- **No springs**, no tendons, no joint torques.
- **At most one leg on the ground** (no double support).

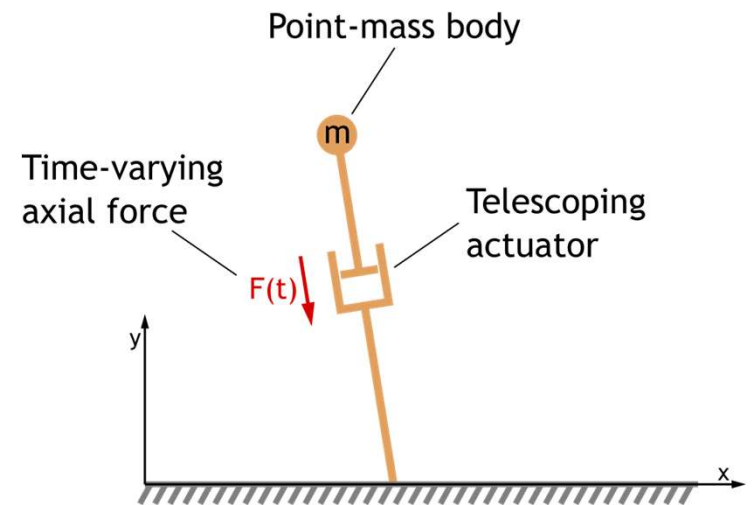
Hypothesis

- **No slip**, foot is fixed during stance.
- Ground is **rigid and flat**.
- Motion alternates between **stance (gravity + force)** and **flight (gravity only)**.
- **Only energetic cost** = positive mechanical work.
- Step length and average speed **fixed for each optimization**.
- Force profile modeled as **piecewise-linear**.
- No sensor and no high-level control

Model & Approach

- The body is a **point mass**
- Legs are **massless** telescopic actuators
- Legs apply a time-varying **axial force** $F(t)$.
- **Objective:** minimize the *mechanical cost over one step* :

$$C = \frac{W}{mgd} = \frac{1}{mgd} \int_0^{t_{\text{step}}} [F(t) \dot{l}]_+ dt$$
- **Constraints:** fixed average speed V , step length D , F Between F_{\min} and F_{\max}
- Solved with **Sequential Quadratic Programming (SQP)** optimization.



Optimisation

$$C = \frac{W}{mgd} = \frac{1}{mgd} \int_0^{t_{\text{step}}} [F(t) l]_+ dt$$

Fixed variables :

- Step length $D = d/l_{\text{max}}$
- Average speed $V = v/\sqrt{gl_{\text{max}}}$
- Maximum leg length l_{max}

Optimized Variables :

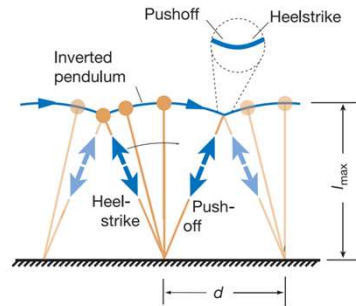
- Leg force profile over time $\bar{F}(\tau)$
 - Initial state of the COM $(X_0, \dot{X}_0, Y_0, \dot{Y}_0)$
 - Stance duration τ_s
- Optimisation through Sequential Quadratic Programming (SQP)
 - Non dimensionalization of all the Variables :

• $X_0 = x_0/l_{\text{max}}$	• $\dot{X}_0 = \dot{x}_0/\sqrt{gl_{\text{max}}}$
• $\bar{F}(\tau) = F(t)/mg$	• $\tau_s = t_s\sqrt{g/l_{\text{max}}}$

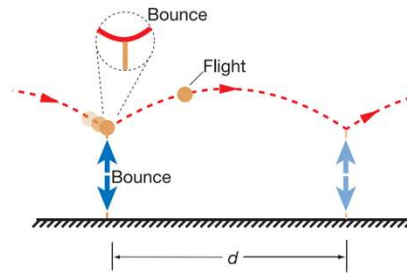
Main Results

Three optimal gaits naturally emerge:

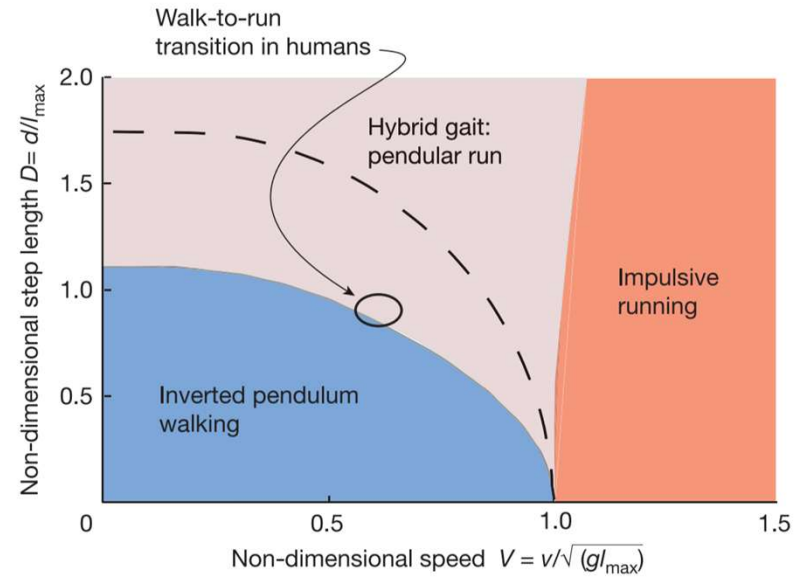
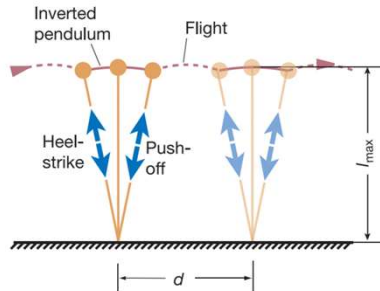
- Pendular walking :



- Impulsive running :

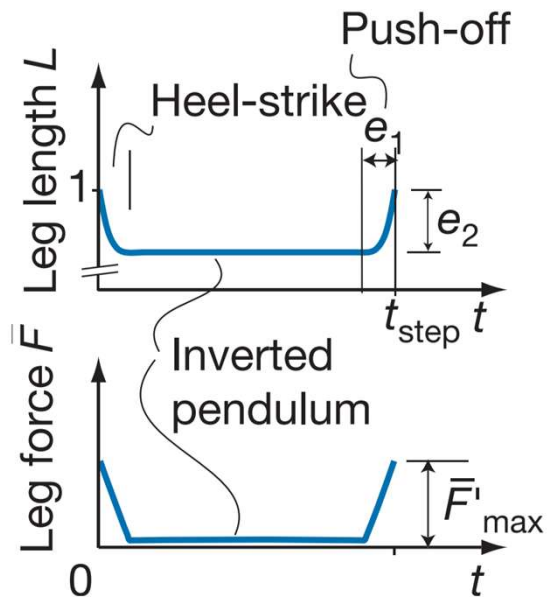


- Pendular running :



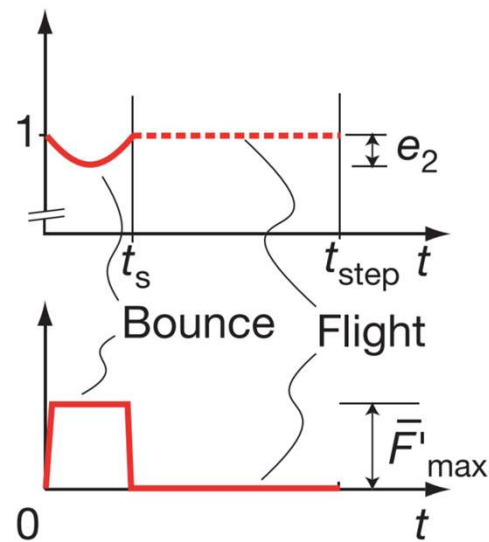
1) Pendular Walk

Center of mass follows a circular arc.



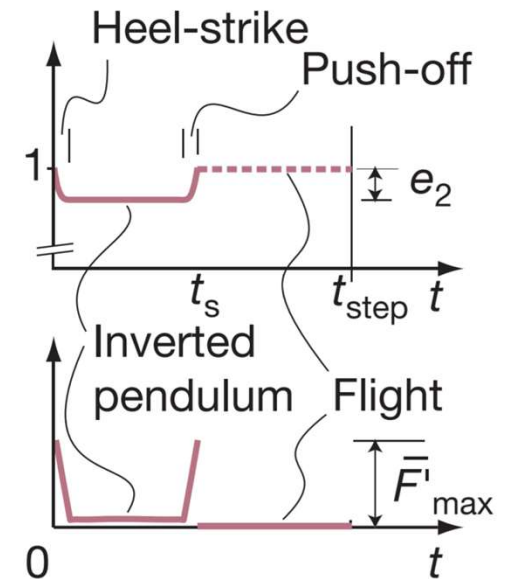
2) Impulsive Run

Aerial phases between ground impulses.



3) Pendular Run

Intermediate gait combining both pendular and flight phases.





Influence of the article

- Google scholar : 503 citations
- Scopus : 5 citations

Influence in robotic research

- Demonstrated that **walking, running, and pendular running** naturally emerge as **energy-optimal gaits** → widely reused to justify gait selection in robotics.
- Became a **foundational reference** for reduced-order models (“template models”) used to describe biped locomotion.
- It has a strong **correlation** with energy optimization for quadrupedal robot across various locomotion modes.
- Used as a **reference model** to generate dynamically consistent trajectories in complex robots.
- Impact not only on robotic, ambulation and sports.

Advantages and disadvantages

Advantages

- Thanks to its simplicity, the model is highly general and can therefore be reused and adapted for many other applications.
- There are no free parameters in the model, everything is fully non-dimensionalized.
- It provides a computed, mechanical explanation of human locomotion energy costs.

Disadvantages

- The model neglects crucial anatomical elements (e.g., tendons, elasticity, real muscle behavior).
- The model does not account for any energy cost during leg swing or stance, and the legs are assumed massless.

**Questions &
Feedback?**

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